Sequential Kriging Metamodel Based Stochastic Global Optimization for TEAM Workshop Problem 22

Su-gil Cho, Minuk Lee and Tae Hee Lee
Department of Automotive Engineering, School of Mechanical Engineering, Hanyang University
Seoul, 133-791, Korea
thlee@hanyang.ac.kr

Abstract — Superconducting magnetic energy storage (SMES) systems store energy in the magnetic field created by the flow of direct current in a superconducting coil. In SMES systems, configurations of coils are regarded as important factors. This problem has been accepted as the TEAM workshop problem 22, which is multi-objective problem with many design variables and constraints. Thus, it is not efficient to apply gradient-based local optimization for this problem. In this paper, sequential kriging metamodel based stochastic global optimization is proposed to resolve these difficulties. Since this technique explores a global optimum by kriging metamodel that can evaluate responses rapidly, computational cost of global optimization can be reduced.

I. INTRODUCTION

SMES devices offer the opportunity to store energy in magnetic fields in a simple way [1]. In SMES solenoid, two concentric coils carrying current with opposite direction and running under superconducting conditions offer the opportunity to store a significant amount of energy in their magnetic fields while keeping the stray field within certain limits. An optimal design of the system should, therefore, couple the desired value of energy to be stored with a minimal stray field. This problem has been accepted as benchmark problem TEAM problem 22 [2].

Design optimization technique becoming increasingly significant in design of industrial products that aim to maximize a desirable performance while satisfying design constraints. Especially, global optimization technique has gained much attention in design of industrial products because it can find not only systematically the best design solution within design space, but also as well as provide multiple alternatives. However, considerable computational burden of global optimization has been still a challenging problem. In order to overcome this difficulty, metamodel has been recently emerged as an efficient alternative [3]. Metamodel is an approximate model to transfer the implicit relationship between design variables and response into explicit one that can be easily expressed with basis functions or polynomials. One of the advantages is that metamodel can evaluate responses quickly.

In this paper, sequential kriging metamodel based stochastic global optimization technique is performed for SMES solenoids problem.

II. SMES PROBLEM

Fig. 1 shows the eight design parameters of two coils. Table I summarizes the constraints on the eight parameters. The objective function of this problem has to take both the energy requirement (E should be as close as possible to 180 MJ) and the stray field requirement (B_{stray} evaluated along 22 equidistant points along line a and line b in Fig. 1 as small as possible) into account; hence, the problem is a multi-objective problem. However, the two objectives are mapped into a single objective function as follows.

$$OF = \frac{B_{stray}^2}{B_{norm}^2} + \frac{\left|E - E_{ref}\right|}{E_{ref}} \tag{1}$$

where E_{ref} = 180MJ, B_{norm} = 200μT and B^2_{stray} is defined as

$$B_{stray}^2 = \frac{\sum_{i=1}^{22} \left|\vec{B}_{stray,i}\right|^2}{22} \tag{2}$$

The superconducting material should not violate the quench condition that links together the value of the current density and the maximum value of magnetic flux density. The critical curve has been approximated by (3).

$$|\mathbf{J}| = \left(-6.4|\mathbf{B}| + 54.0\right) A/mm^2 \tag{3}$$

This optimization problem has a constraint, 8-design variables and nonlinear objective function that is consist of error of energy to be stored and stray field. It is not easy to find optimum solution of this problem. Therefore, in the next part, proposed global optimization is explained in order to resolve the difficulty

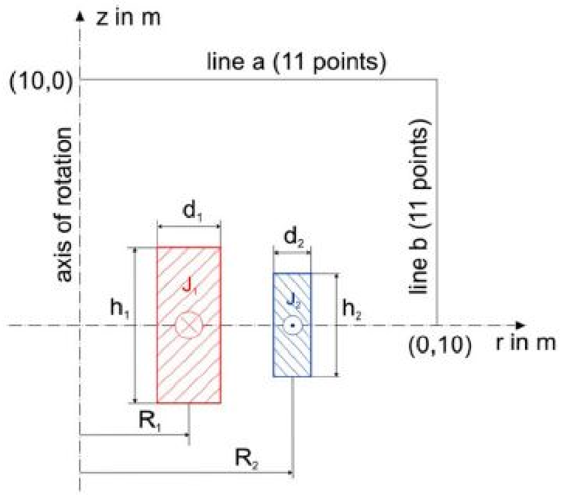

Fig. 1. Configuration of the SMES device with eight parameters

TABLE I
BOX CONSTRAINTS OF THE SMES PROBLEM

limits	R_1 $[m]$	R_2 $[m]$	$h_1/2$ $[m]$	$h_2/2$ $[m]$	d_1 $[m]$	d_2 $[m]$	J_1 $\left[\frac{MA}{m^2}\right]$	J_2 $\left[\frac{MA}{m^2}\right]$
lower	2.00	2.60	0.8	0.204	0.27	0.1	22.5	-22.5
upper		3.40		1.1		0.4		

III. SEQUENTIAL KRIGING METAMODEL BASED STOCHASTIC GLOBAL OPTIMIZATION

Kriging, so-called design and analysis of computer experiment (DACE) model, is the interpolation model where the prediction coincides with the simulation response at sampled points exactly [4]. Once the kriging metamodel has been generated, it can provide not only the predicted value, but also its stochastic prediction error denoted by $\hat{\sigma}^2$, the so-called mean squared error(MSE). MSE is directly related to the uncertainty of the predicted value. Let us consider a response of kriging metamodel $\hat{Y}(x_1)$ at design point of x_1. In kriging theory, there is a basic assumption that both sample data and predicted values of kriging metamodel are normally distributed. The stochastic probability that $\hat{Y}(x_1)$ is smaller than $f_{\min}$, the probability of improvement, is defined as

$$P\left(\hat{Y}(x_1) \le f_{\min}\right) = \Phi(I_1) = \Phi\left(\frac{f_{\min} - \hat{Y}(x_1)}{\hat{\sigma}_Y(x_1)}\right) \tag{4}$$

where $\Phi(\bullet)$ is CDF of standard normal distribution and I_1 is a stochastic quantity depending on x_1. $f_{\min}$ denotes the minimum value among observed data.

On the other hand, if we consider G(x) as a constraint function, the probability of feasibility can be defined just like probability of improvement. Similar to (4), the probability of feasibility at x_1 is defined as follows

$$\Phi(F_1) = \Phi\left(\frac{0 - \hat{G}(x_1)}{\hat{\sigma}_G(x_1)}\right) \tag{5}$$

Sequential kriging metamodel based stochastic global optimization technique has the following search strategies.

Step 1: An initial sample set is chosen and then kriging metamodels for objective and constraint functions are constructed.

Step 2: If no feasible point has yet been sampled, explore a feasible point using probability of feasibility,

$$maximize \prod_{j=1}^{n_c} \Phi\left(\frac{0 - \hat{G}_j(\mathbf{x})}{\hat{\sigma}_{G_j}}\right) \tag{6}$$

otherwise go to step 3.

Step 3: Solve an original constrained optimization with metamodels of objective and constraint functions as follows:

$$\begin{aligned} &minimize\ \hat{Y}(\mathbf{x}) \\ &subject\ to\ \hat{G}_j(\mathbf{x}) \le 0 \left(j = 1,2,\ldots,n_c\right) \end{aligned} \tag{7}$$

If at least three points have already been sampled within tol_1 of the last iteration, then go to step 4.

Step 4: Attempt to find an additional feasible point using Eq. (8). Once a feasible point is sampled at least tol_2 away from other feasible samples, begin another local search (go to step3).

$$maximize \prod_{j=1}^{n_c} \Phi\left(\frac{0 - \hat{G}_j(\mathbf{x})}{\hat{\sigma}_{G_j}}\right) \cdot \Phi\left(\frac{f_{\min} - \hat{Y}(\mathbf{x})}{\hat{\sigma}_Y}\right) \cdot D \tag{8}$$

Step 5: The process has been terminated once the number of total function calls exceeds a user-defined threshold. In our implementation, we set the parameters tol_1=1% and tol_2=5%, D is minimum distance between sample points.

IV. RESULTS

For applying this algorithm, 3-level full factorial sampling is used as initial sample set. The total number of function calls is specified as 100. As shown in Fig. 2, we can find a global optimum with 84 function calls. The global optimum is $\mathbf{x}_{opt} = [3.11,\ 0.239,\ 0.384]$ and constraint becomes active at optimum. The objective function value of optimum design decrease by about 71.64% respectively compared with initial point, and we can find alternative local optimum. The results are shown in table Ⅱ.

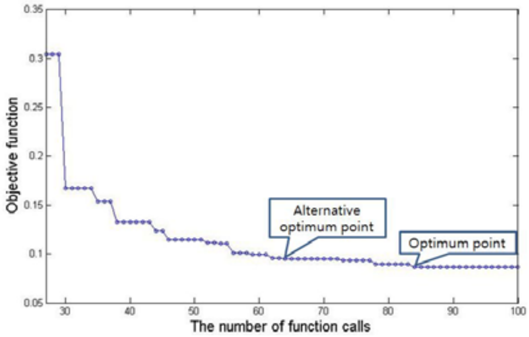

Fig. 2. History plot of optimization

TABLE Ⅱ
OPTIMIZATION RESULT

Optimum point	B^2_{stray} [μT]	Energy [MJ]	Object function	Function calls for optimum
[3.11, 0.239, 0.384]	0.776	179.97	0.0864	84
[3.17,0.357,0.248]	0.846	179.61	0.0962	

V. CONCLUSION

TEAM workshop problem 22 is multi-objective problem with many design variables and constraints. To resolve this problem, it is required to apply global optimization. However, typical global optimization techniques need a lot of function calls. In this paper, sequential metamodel based statistical global optimization technique is proposed. In this proposed procedure, we can search for an initial feasible region and refine a local optimum. Then, we explore another feasible region with the highest probability of improving upon the current best point. Therefore, computational cost of global optimization can be significantly reduced. Also we can find alternative local optima.

VI. REFERENCES

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